Hidden Symmetry and S-duality in N=4 D=4 Super Yang-Mills Theory

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Abstract

In this talk the Schwarz hypothesis that the duality symmetries should be pieces of the hidden gauge symmetry in a string theory is discussed. Using auxiliary linear system special dual transformations for $N = 4$ SYM generalizing the Schwarz dual transformations for a principal chiral model are constructed. These transformations are related with a continuous group of hidden symmetry of a new model involving more fields as compare with $N = 4$ SYM. We conjecture that the Z_2 discrete subgroup of this hidden symmetry group has a stable set of N=4 YM fields and transforms a self-dual configuration to an anti-self-dual and via versa.

1 Introduction

In the recent years, the idea that string theory will exhibit an enormous gauge symmetry group has got many supports. There are the following indications supporting this idea.

• More then ten years ego it was realized that the theories which now are considered as describing a low energy limit of string theories do posses some hidden symmetries. Namely, there are global continuous non-compact symmetries in classical supergravity theories. A non-compact $SL(2, R)$ symmetry group was found in $N = 4,\ D = 4$ supergravity by Cremer, Ferrara and Scherk [2] and E_7 symmetry was found in $N = 8$, $D = 4$ supergravity by Cremer and Julia [3] (about other examples see [4]). More yearly Geroch [5] has found special solutions of Einstein's equations that were invariant under an infinite set of transformations (about further developments see [6] and about applications to strings see [7]).

• The existence of the Lax representation for $N = 4$ $D = 4$ and $N = 1$, $D = 10$ super Yang-Mills theories [8] was associated, due to wide experience in two-dimensional integrable models [9], with some hidden symmetry. For these models there are infinite number of conserved currents [10, 11] generalized the Lucher-Pohlmeyer-Brezin et al. currents [12, 13] of two-dimensional integrable models. In the two-dimensional cases current conservations provide enough restrictions in order to solve the theory exactly.

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• A search for a symmetry in string theory was one of motivation of the Witten construction of the string field theory [15].

• The idea of enormous symmetry in string theory was advocated by Gross and Mende in the context of high energy scattering of strings [14].

• This idea got essentially more support after discovering the duality symmetry in string theory. Few years ego, Font, Inanez, Lust and Quevedo [16] proposed that an $SL(2,Z)$ discrete subgroup of SL(2,R) should be an exact symmetry of the heterotic string toroidally compactifies to four dimensions (see also [17, 18]). This duality symmetry in a special case implies an electric-magnetic duality and is intrinsically non-perturbative and therefore cannot be test within perturbation theory. Initial evidence in favour of s-duality for N=4 SYM was provided by the exact agreement of masses of particles and solitons with those predicted by the Z_2 symmetry of Montenen and Olive. Tests of full $SL(2,Z)$ s-duality have been formulated and verified in [19, 20]. Girardello et al [19] and Vafa and Witten [20] have examined the implications of s-duality for N=4 SYM, and Seiberg and Witten [21] have examined N=2 SYM.

• There is the Schwarz conjecture [1] that the duality symmetries should be pieces of the hidden gauge symmetry in a string theory. The string duality symmetries are infinite discrete groups. The conjecture is that they are the discrete subgroups of gauge symmetries of string theory. Note that symmetry groups appears in the classical supergravity are continuous ones. At first sight there is a discrepancy between continuous symmetry of classical theories and discreteness of the group of duality transformations. One can expect that the restrictions to discrete groups are due to quantum corrections or sting effects. In particular, Hull and Townsend [22] have shown that E_7 symmetry of N=8, D=4 supergravity found in [3] is broken by quantum effects to a discrete subgroup, $E_7(Z)$, which contains both the T-duality group $O(6,6,Z)$ and the S-duality group $SL(2,Z)$.

But another point of view is also admissible. Apparently, there is no raison to expect that classical and quantum properties of N=4, D=4 , which describes a low energy sector of string theory after trivial reduction, are strongly different. The validity of this opinion relies on the existence of non-renormalization theorems. In the example considered by Hull and Townsend non-renormalization theorem is not valued. If so, it is not worth to try to find an continuous hidden symmetry group for N=4 SYM. Note that in spite of existence of conserved currents the hidden symmetry for N=4, D=4 super Yang-Mills is still not found. We will see on the model example that the existence of the continuous duality group can be depended on the boundary conditions of the model under consideration, or on a coefficient in front of an topological term.

We are going to start from a consideration the Schwarz dual transformation for the D=2 principal chiral model with anomaly term. Note that since duality is an intrinsically non-perturbative property it is appropriate in this context to use our knowledge concerning D=2 exact integrable models [9]. We will see that for all possible θ except $\theta = 1$, it is possible to do dual transformation, parametrized by some parameter λ, $-(1+\theta) < \lambda < 1+\theta$ getting a new θ, say θ' according the formula

$$\theta' = \frac{\lambda - \theta(1+\theta)}{\lambda\theta - (1+\theta)}. \tag{1.1}$$

We see from this formula that $\theta = \pm 1$ are the stable points of this transformation. In these special points one can make another dual transformation which makes a change

$$\theta' = -\theta \tag{1.2}$$

This gives an example then the continuous group in special values of parameters of the model under consideration shrink to discrete Z_2 group.

We expect the similar picture for $N = 4$, $D = 4$ being embedded in a more large model. In particular, there is so called N=4, D=4 B-model which as compare with N=4 SYM contains one extra dynamical field B. For $B = 0$ this model coincides with SYM. This model is non-relativistic one, but it is supersymmetric and gauge invariant. This model possesses a hidden symmetry. Applying infinitezimally these symmetry transformations to N=4 SYM we get non-zero value for field B and we abandon mass-shell of N=4 SYM and occur on mass-shell of B-model. But it may happen that after some global transformation we will come back to B=0. In this case one can say that continuous group of the large model (B-model in our case) is reduced to a discrete subgroup of more restricted model, i.e. model which has more constrains (N=4 SYM in our case). It is tempting to expect that a transformation which transforms a self-dual configuration to an anti-self-dual one for SYM model can be considered as a global hidden symmetry transformation for B-model.

2 Dual Transformations for Principal Chiral Models

An equation of motion for a principal chiral model (PCM) with an anomaly (a WZNW term) has the form

$$\partial_\mu A_\mu + \theta \epsilon_{\mu\nu} \partial_\mu A_\nu = 0, \tag{2.1}$$

where

$$A_\mu = g^{-1} \partial_\mu g, \tag{2.2}$$

so A_μ satisfies also the zero-curvature condition

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \tag{2.3}$$

It is convenient to introduce light-cone coordinates

$$x^\pm = x^0 \pm x^1, \partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1), \tag{2.4}$$

so that the equation of motion takes the form

$$\partial_+ A_- + \kappa \partial_- A_+ = 0. \tag{2.5}$$

Let us consider the Schwarz dual transformation [1]

$$A_\mu \ \rightarrow \ A'_\mu = g'^{-1} \partial_\mu g', \tag{2.6}$$

where

$$g' = g(\theta, x) \Psi(\theta, \lambda), \tag{2.7}$$

$g(\theta, x)$ is a solution of (2.1) and $\Psi(\theta, \lambda)$ is a solution of the following linear system of equations

$$(\partial_+ - \frac{\lambda(1-\theta)}{1+\theta-\lambda} A_+(\theta, x))\Psi(\theta, \lambda) = 0, \tag{2.8}$$

$$(\partial_- + \frac{\lambda(1+\theta)}{1+\theta+\lambda} A_-(\theta, x))\Psi(\theta, \lambda) = 0, \tag{2.9}$$

$$-(1+\theta) < \lambda < 1+\theta. \tag{2.10}$$

Simple calculations show that after the dual transformation of the field $g(c,x)$ we get once again the chiral field with a new coefficient in front of the WZNW term. Namely, one can check that

$$\partial_\mu A'_\mu(\theta,x) + \theta' \epsilon_{\mu\nu} \partial_\mu A'_\nu(\theta,x) = 0 \tag{2.11}$$

with

$$\theta' = \frac{\lambda - \theta(1+\theta)}{\lambda\theta - (1+\theta)}. \tag{2.12}$$

The function (2.12) has the following interesting property: starting from $\theta = \pm 1$ after the dual transformation with arbitrary λ we get $\theta' = \pm 1$, i.e *the point $\theta = \pm 1$ is stable under the dual transformation.*

As Schwarz pointed out [1] starting from $\theta = 0$ after the dual transformation with a suitable λ one can get any $-1 < \theta' < 1$. The same is true for arbitrary $\theta \neq \pm 1$. In particular, starting from $\theta \neq 1$ after the dual transformation with a suitable λ (which is very closed to the boundary $\lambda = -(1+\theta)$) one can get θ' which is very closed to 1, but never we can get exactly $\theta' = 1$ if $\theta \neq 1$. Formally, to get exactly $\theta' = 1$ we have to make the dual transformation with $\lambda = -(1+\theta)$. However just this point is a singular point for linear equations (2.9). So we can say that in the points $\theta = \pm 1$ there are isolated "traps".

The same picture takes place for a superchiral field. Equation of motion for the superchiral field with the anomaly (the WZNW term) has the form

$$D_1 A_2 - \kappa D_2 A_1 = 0, \tag{2.13}$$

where A_i is defined as

$$A_i = G^{-1}(y) D_i G(y). \tag{2.14}$$

Here $G(y)$ is a superfield

$$G(y) = \exp \sum_a T_a \phi_a(x_\mu, \theta_1, \theta_2), \quad y_A = (\mu, \theta_1, \theta_2),$$

$$D_1 = \frac{\partial}{\partial\theta_2} - i\theta_2 \frac{\partial}{\partial x^+}, \quad D_2 = -\frac{\partial}{\partial\theta_1} + i\theta_1 \frac{\partial}{\partial x^-}, \tag{2.15}$$

x^+ and x^- are light-cone coordinates (2.4). Supercurrent A_i also satisfies the zero-curvature identity

$$D_1 A_2 + D_2 A_1 + \{A_1, A_2\} = 0. \tag{2.16}$$

Compatibility conditions for the following linear system [23]

$$D_1 \Psi(\lambda) = \frac{\lambda\kappa}{1 - \lambda \frac{1+\kappa}{2}} A_1 \Psi \tag{2.17}$$

$$D_2 \Psi(\lambda) = -\frac{\lambda}{1 + \lambda \frac{1+\kappa}{2}} A_2 \Psi. \tag{2.18}$$

are just equations (2.13) and (2.16). To recall that $G(y)$ is a solution of equation (2.13) with some κ we write $G_\kappa(y)$ and $\Psi_\kappa(\lambda)$.

Now let do an analog of the transformation (2.7). Multiplying $G_\kappa(y)$ on $\Psi_\kappa(\lambda)$

$$G' = G_\kappa \Psi_\kappa(\lambda) \tag{2.19}$$

one can check that

$$A'_i = G'^{-1}(y) D_i G'(y) \tag{2.20}$$

satisfies the following equation

$$D_1 A'_2 - \kappa' D_2 A'_1 = 0, \tag{2.21}$$

where

$$\kappa' = \kappa \frac{1+\lambda^{\frac{1+\kappa}{2}}}{1-\lambda^{\frac{1+\kappa}{2}}} \tag{2.22}$$

And once again we have stable points of the dual transformation. They are $\kappa = 0$ and $\kappa = \infty$

3 Dual Transformations for N=4 D=4 Super Yang-Mills Theory

For N=4 SYM it is possible to do an analog of the Schwarz dual transformation (2.7). Namely, starting form superpotential which is a solution of the self-dual equations one can construct a superpotential which is a solution of the anti-self-dual equations. This can be consider as an analog of transformations which changes the sign in the front of the WZNW term. To do this I'l use a linear system of equations which compatibility conditions coincide with full SYM equation [8].

Let $C^{4,4N}$ be the complex Minkowski superspace with coordinates

$$y^A = (x^\mu, \theta^\alpha_s, \bar{\theta}^{\dot{\alpha}t}), \quad \mu = 0,1,2,3; \quad \alpha, \dot{\beta} = 1,2; \quad s,t = 1,...N = 4$$

Covariant derivatives have the form

$$\mathcal{D}_A = D_A + [A_A, \ . \ ],$$

where

$$D_A = (\partial_\mu, D^s_\alpha, D_{\dot{\beta}t}), \ D^s_\alpha = \frac{\partial}{\partial \theta^\alpha_s} + i\bar{\theta}^{\cdot \beta s}\partial_{\alpha \cdot \beta}, \quad D_{\dot{\beta}t}), \quad D_{\dot{\beta}t} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\beta}t}} - i\theta^\alpha_t \partial_{\alpha\dot{\beta}} \partial_{\alpha \cdot \beta}.$$

Supercurvature F_{AB} is defined as

$$[\mathcal{D}_A, \mathcal{D}_B] = T^C_{AB}\mathcal{D}_C + iF_{AB}, \tag{3.1}$$

where T^C_{AB} is a torsion,

$$T^s_{\alpha,\dot{\beta}t}{}^\mu = T^s_{\dot{\beta},t\alpha}{}^\mu = -2i\delta^s_t \sigma^\mu_{\alpha \cdot \beta}$$

The equations of motion have the form

$$F^{(st)}_{\alpha\beta} = 0 = F_{\dot{\alpha}(s,\dot{\beta}t)} \tag{3.2}$$

$$F^s_{\alpha,\dot\beta t} = 0 \tag{3.3}$$

These equations look as the constraint equations, however it is known that they lead to differential equations in x-space (put the theory on-shell) [24, 25, 8],[26]-[29].

Let consider a set of linear equations [8]

$$X^s(\lambda)\Psi(\lambda) = 0, \tag{3.4}$$

$$Y_t(\lambda)\Psi(\lambda) = 0, \tag{3.5}$$

$$Z(\lambda)\Psi(\lambda) = 0, \tag{3.6}$$

where $X^s(\lambda), Y_t(\lambda)$ and $Z(\lambda)$ are given by the following formulae

$$X^s(\lambda) = \nabla^s_1 + \lambda\nabla^s_2,$$

$$Y_t(\lambda) = \nabla_{\dot 1 t} + \lambda^2\nabla_{\dot 2 t},$$

$$Z(\lambda) = \nabla_{1\dot 1} + \lambda\nabla_{2\dot 1} + \lambda^2\nabla_{1\dot 2} + \lambda^3\nabla_{2\dot 2}.$$

The integrability conditions of these linear equatioins are

$$\{X^s(\lambda), X^t(\lambda)\} = 0 = \{Y_s(\lambda), Y_t(\lambda)\}, \tag{3.7}$$

$$\{X^s(\lambda), Y_t(\lambda)\} = -2i\delta^s_t Z(\lambda) \tag{3.8}$$

One can also write a "covariant" linear system

$$z^\alpha\nabla^t_\alpha\Psi(z,w) = 0 \tag{3.9}$$

$$w^{\dot\alpha}\nabla_{\dot\alpha t}\Psi(z,w) = 0 \tag{3.10}$$

$$z^\alpha w^{\dot\alpha}\nabla_{\alpha\dot\alpha}\Psi(z,w) = 0 \tag{3.11}$$

One gets (3.4), (3.5) and (3.6) from (3.9), (3.10) and (3.11) substituting

$$z^1 = 1,\ z^2 = \lambda,\ \ w^{\dot 1} = 1,\ w^{\dot 2} = \lambda^2 \tag{3.12}$$

Equations of motion (3.2) and (3.3) provide the consistency conditions for the linear system (3.9)-(3.11).

Equations which generalize the self-dual equations of the usual Yang-Mills theory in the superfield notations have the form ([30], see also [31])

$$F^{st}_{\alpha\beta} = 0 = F_{\dot\alpha(s,\dot\beta t)} \tag{3.13}$$

$$F^s_{\alpha,\dot\beta t} = 0 \tag{3.14}$$

For the anti-self-dual case one has

$$F^{(st)}_{\alpha\beta} = 0 = F_{\dot\alpha s,\dot\beta t} \tag{3.15}$$

$$F^s_{\alpha,\dot\beta t} = 0 \tag{3.16}$$

There is no symmetrization on the upper s and t in (3.13) and lower s and t in (3.15).

One has the following linear system for the selfdual super-Yang-Mills equations

$$\nabla^s_\alpha\Psi^{\mathcal{SD}}(w) = 0 \tag{3.17}$$

$$w^{\dot\alpha}\nabla_{\dot\alpha t}\Psi^{\mathcal{SD}}(w)=0 \tag{3.18}$$

$$w^{\dot\alpha}\nabla_{\alpha\dot\alpha}\Psi^{\mathcal{SD}}(w)=0 \tag{3.19}$$

and the following linear system for the anti-self-dual Super-Yang-Mills equations

$$z^{\alpha}\nabla^{s}_{\alpha}\Psi^{\mathcal{ASD}}(z)=0 \tag{3.20}$$

$$\nabla_{\dot\alpha t}\Psi^{\mathcal{ASD}}(z)=0 \tag{3.21}$$

$$z^{\alpha}\nabla_{\alpha\dot\alpha}\Psi^{\mathcal{ASD}}(z)=0 \tag{3.22}$$

Starting form superpotential $(A^{\mathcal{SD}}{}_{\alpha}{}^{s}, A^{\mathcal{SD}}_{\dot\beta t}, A^{\mathcal{SD}}_{\alpha\dot\beta})$ being solution of the self-dual equations (3.13) and (3.14) let construct a solution of anti-self-dual equations (3.15) and (3.16). To do this let consider the solution of the linear system of equations (3.4), (3.5) and (3.6) for given superpotential (compatibility condition takes place since this superconnection solves (3.2) and (3.3).

So we have $\Psi(z,w|A^{\mathcal{SD}}_{A})$. Let now take

$$A'^{s}_{\alpha}=A^{\mathcal{SD}s}_{\alpha}, \tag{3.23}$$

$$A'_{\dot 1 t}=A^{\mathcal{SD}}_{\dot 1 t} \tag{3.24}$$

$$A'_{\dot 2 t}=A^{\mathcal{SD}}_{\dot 2 t}-[(D_{\dot 2}+A^{\mathcal{SD}}_{\dot 1 t})\Psi_1]\cdot\Psi_1^{-1} \tag{3.25}$$

where

$$\Psi_1\equiv\Psi(z,w|A^{\mathcal{SD}}_{A})|_{\substack{w^{\dot 1}=1\\ w^{\dot 2}=0}} \tag{3.26}$$

Now one can check that the following equations are fulfilled

$$z^{\alpha}\nabla'^{s}_{\alpha}\Psi_1(z)=0 \tag{3.27}$$

$$\nabla'_{\dot\alpha t}\Psi_1(z)=0 \tag{3.28}$$

where $'$ means

$$\nabla'_{\dot\alpha t}=D_{\dot\alpha t}+A'_{\dot\alpha t}$$

Vectors components can be obtained from the spinors components of the superconnection. Transformation (3.25) is a generalization of the Schwarz transformation (2.6).

4 B-model

Let us consider a model described by the superconnection $A_A(y)$ and an additional superfield $B_s(y)$ [32]. These fields satisfy the following dynamical equations

$$F^{(st)}_{\alpha\beta}=0,\quad F_{\dot 1 s,\dot 1 t}=0=F_{\dot 2 s,\dot 2 t} \tag{4.1}$$

$$F_{\dot 1(t,\dot 2 s)}+B_{(t}B_{s)}=0, \tag{4.2}$$

$$\mathcal{D}_{\dot\alpha(t}B_{s)}=0,\quad F^{s}_{1,\dot 1 t}=0=F^{s}_{2,\dot 2 t}, \tag{4.3}$$

$$F^{s}_{2,\dot 1 t}+\mathcal{D}^{s}_{2}B_t=0 \tag{4.4}$$

This system of equations is non-relativistic one. But it is invariant under the gauge transformation

$$A_A \to K^{-1}A_AK + K^{-1}D_AK, \quad B \to K^{-1}BK \tag{4.5}$$

where K is an arbitrary superfield. If $B = 0$ equations (4.1)-(4.4) coincide with (3.2)-(3.3)

Let consider a set of linear systems

$$\mathcal{X}^s(\lambda)\Psi(\lambda) = 0, \tag{4.6}$$

$$\mathcal{Y}_t(\lambda)\Psi(\lambda) = 0, \tag{4.7}$$

$$\mathcal{Z}(\lambda)\Psi(\lambda) = 0, \tag{4.8}$$

where $\mathcal{X}^s(\lambda), \mathcal{Y}_t(\lambda)$ and $\mathcal{Z}(\lambda)$ are given by the following formulae

$$\mathcal{X}^s(\lambda) = \nabla_1^s + \lambda\nabla_2^s,$$
$$\mathcal{Y}_t(\lambda) = \nabla_{\dot{1}t} + \lambda B_t + \lambda^2\nabla_{\dot{2}t},$$
$$Z(\lambda) = \nabla_{1\dot{1}} + \lambda\nabla_{2\dot{1}} + \lambda^2\nabla_{1\dot{2}} + \lambda^3\nabla_{2\dot{2}}.$$

We see that $\mathcal{X}$ and $\mathcal{Z}$ coincide with X and Z respectively, and only there is a difference in the operators $\mathcal{Z}$ and Z.

The integrability conditions of these linear equatioins,

$$\{\mathcal{X}^s(\lambda), \mathcal{X}^t(\lambda)\} = 0 = \{\mathcal{Y}_s(\lambda), \mathcal{Y}_t(\lambda)\}, \tag{4.9}$$

$$\{\mathcal{X}^s(\lambda), \mathcal{Y}_t(\lambda)\} = -2i\delta_t^s\mathcal{Z}(\lambda), \tag{4.10}$$

are provided by equations (4.1)-(4.4).

We can write the following hidden symmetry transformation for the system equations (4.1)-(4.4).

$$\delta A_1^s = 0, \quad \delta A_{\dot{1}s} = 0, \quad \delta A_{1\dot{1}} = 0, \tag{4.11}$$

$$\delta A_2^s = -\epsilon i\mathcal{D}_2^s S(\mu), \tag{4.12}$$

$$\delta A_{\dot{2}t} = -\epsilon i\mathcal{D}_{\dot{2}t}S(\mu), \tag{4.13}$$

$$\delta A_{2\dot{2}} = -\epsilon i\mathcal{D}_{2\dot{2}}S(\mu), \tag{4.14}$$

$$\delta A_{2\dot{1}} = -\epsilon i\frac{1}{\mu}(\partial_{1\dot{1}}S(\mu), \tag{4.15}$$

$$\delta A_{1\dot{2}} = -\epsilon i(\mathcal{D}_{1\dot{2}}S(\mu) + \mu\mathcal{D}_{2\dot{2}}S(\mu)), \tag{4.16}$$

$$\delta B_t = i\mu\delta A_{\dot{2}t} + \epsilon[B_t, S(\mu)], \tag{4.17}$$

where $S(\mu)$ is a solution of the following equations

$$(D_1^s + \mu\mathcal{D}_2^s)S(\mu) = 0, \tag{4.18}$$

$$(D_{\dot{1}t} + \mu^2\mathcal{D}_{\dot{2}t})S(\mu) = -\mu[B_t, S(\mu)], \tag{4.19}$$

$$(\partial_{1\dot{1}} + \mu\mathcal{D}_{2\dot{1}} + \mu^2\mathcal{D}_{1\dot{2}} + \mu^3\mathcal{D}_{2\dot{2}})S(\mu), \tag{4.20}$$

We see that transformations (4.11), (4.13) are nothing but an infinitezimal version of (3.23)-(3.25).

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